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LETTER TO THE EDITOR

Telescope phasing and ground states of solid-on-solid models

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Abstract. Wavefront phase perturbations are a source of severe image degradation in large astronomical telescopes. This degradation can, in principle, be compensated by suitable surface corrections in a multi-mirror telescope. We propose an analogy between such a telescope and a solid-on-soil model for a crystal surface. Mirrors correspond to columns of atoms, and optimal phasing corresponds to a flat crystal surface. Simulation of an annealing process that gives flat crystal surfaces at low temperatures is found to provide a reliable algorithm for phasing the telescope. Annealing, suitably combined with the use of several wavelengths, avoids local minima of the effective Hamiltonian for the intensity pattern.

A severe problem facing ground-based astronomical observations is the image degradation which results from atmospheric turbulence [1]. Active systems have been proposed, essentially based on a multi-faceted mirror concept, by which the distorted incident wavefront is corrected by suitably adjusting the relative phases of the waves reflected from the individual facets. No great success has yet been reported for such systems. A multi-mirror space telescope will be expected to suffer from essentially similar wavefront degradation because of mechanical distortion introduced during launching [2, 3].

In searching for a practical way by which the phase correction can be carried out, we have considered in detail methods which rely only on data that can be extracted from the image itself [4–6]. One way which has already been suggested [7] uses an algorithm which searches for the sharpest image of a point source by making random changes to the positions of the mirror facets and accepting all changes which improve the sharpness, according to some defined criterion. However since the distorted image is speckled, such an algorithm usually leads into a local optimum, which is not the diffraction-limited image for the complete telescope but nevertheless deteriorates whenever change is made to any facet.

In this letter we propose a novel solution to this telescope phasing problem, motivated by an analogy between the optimal reflected wavefront and the ground state of a solid-on-solid (SOS) model. The SOS model [8] mimics the arrangement of atoms on a crystal surface; these atoms arrange as a flat faceted crystal surface at low temperatures but form a rough surface with columns of varying heights at higher temperatures. The adjustable sub-elements of the telescope correspond to the columns

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of atoms, and the optimal phasing of these elements corresponds to the crystal surface ground state, which is flat or of uniform slope.

A Hamiltonian for the energy of the sos model on a planar lattice, where there is a column of atoms of height h_k (relative to some arbitrary reference plane) at the position k can be written

$$\mathscr{H} = \sum_{k>1} J_{kl}(h_k - h_l)^2 \tag{1}$$

where J_{kl} is a function of the distance between the sites k and l. The ground state of the system is that choice of h_k which minimises the value of the total $\mathscr{H}$; this will occur when all pairs of height differences are as small as possible.

By analogy, we define an energy ('Hamiltonian') function for the telescope which will be minimised when the telescope is phased (i.e. the wavefront reflected from its aperture is spherical, flat if the image is at infinity). For this Hamiltonian, we choose

$$\mathscr{H} = \sum_{k>l} |\Gamma_{kl}|^2 \left[1 - \cos\left(\frac{2\pi}{\lambda}(h_k - h_l)\right)\right]. \tag{2}$$

The wavelength of measurement is λ and Γ_{kl} is the mutual coherence function between points k and l on the telescope aperture. In this case h_k is measured from a spherical reference wavefront focused on the image plane. Now the electromagnetic waves at the telescope aperture and at its focal plane are a Fourier pair, in which case Hamaker *et al* [9] showed that minimising the Hamiltonian in (2) is equivalent to maximising Muller and Buffington's sharpness function [7]

$$\mathscr{S} = \sum_m I_m^2 \tag{3}$$

where I_m is the light intensity measured by a detector at position m in the focal (Fourier) plane of the telescope. According to the van Cittert–Zernicke theorem [10], Γ_{kl} is the Fourier transform of the (stellar) object intensity. In a real optical system, it is faster to measure the image sharpness from (3), while (2) is faster to calculate in a computer simulation.

A comparison of (1) and (2) clearly illustrates the analogy between the two systems. J_{kl} corresponds to $|\Gamma_{kl}|^2$, both being functions of the distance between two sites. These both multiply functions of the height differences $h_k - h_l$ which are respectively minimised when the solid state system is in its flat ground state and when the telescope is phased, i.e. the outgoing wavefront is flat, modulo λ. In nature, crystalline ground states are reached by annealing processes, since quenching can lead to metastable, less ordered structures. In computer experiments on the sos model, a simulated annealing process [11–14] can be used to determine the ground-state configuration, by slowly 'cooling' samples from a high-temperature rough phase. This is achieved in the simulation by initially setting the h_k to random values, and then sweeping through the system, and randomly selecting a possible change $\Delta h_k = \pm 1$ for each column. Each change that decreases the Hamiltonian is carried out, whereas changes that could raise the total energy by ΔE are carried out with a probability $e^{(-\Delta E/kT)}$ as required by detailed balance. At each temperature T, the columns are allowed to rearrange into an equilibrium state. In the event that local minima are encountered, corresponding to metastable configurations, the randomness of the annealing process and variations in the sample temperature eventually enable the system to escape [15]. Our analogy suggests that simulated annealing is a promising approach for the determination of the optimal phasing of the wavefront within realistic time spans. This is not the first application of simulated annealing to problems outside condensed matter physics†. However this application is special, both because of the closeness of the analogy and because the visualisation of telescope models as particular varieties of SOS models, leads to some interesting possibilities for study of the latter systems. In the language of our analogy, the approach of Muller and Buffington‡ corresponds to a rapid quench which generally leads into a local minimum.

Local minima are a far more serious problem for the optical system than for simulations of the usual type of SOS model. They were a minor impediment in some past SOS simulations [14] because double steps lead to local metastability, but since the major interest in those calculations was the region near the roughening transition, the problem was bypassed in practice by starting from the ordered state. Since we wish to obtain an algorithm for annealing the system into its ordered state, we must face the ubiquitous local minimum problem directly. The dominant cause of the local minima in the optical system is the periodic nature of light which is implicit in the cosines of (2). A major theme of this letter is our solution to the local minima problem in the optical system; this solution also has exciting implications for the study of glassy phenomena on sloped crystal surfaces§. For a single wavelength of light, there is no unique minimum for $\mathscr{H}$ because height differences of $n\lambda/2\pi$ are indistinguishable. This degeneracy can be resolved by simultaneous introduction of several distinct wavelengths, but the required number of steps to reach optimal phasing was found to be unrealistic. Similarly, very wide band light (white light) proved to be a poor choice, since the mutual coherence function Γ_{kl} is a strongly declining function of optical path difference. In other words, if two panels are not phased to within a few wavelengths, one cannot obtain white light fringes between them, and the intensity pattern (3) remains unchanged.

A novel aspect of our approach is the introduction of a scheduled succession of closely related wavelengths to circumvent the problems caused by the periodicity. At first, the height of the elements is changed by steps whose size (Δh) is chosen to be a simple fraction of the initial wavelength of measurement (λ_0), and the wavefront anneals to a plane surface, modulo the wavelength. For the rest of the annealing the wavelength is varied to be $\lambda_0 \leqslant \lambda \leqslant \lambda_0(1+\Delta h/h_{\max})$, where $h_{\max}$ is the maximum anticipated deviation; at the same time the step size by which the mirrors are moved is enlarged to be the *initial* wavelength, λ_0. Between the two wavelengths we obtain

† Kirkpatrick *et al* [16] pioneered the application of simulated annealing to problems outside statistical physics. A recent application of the method to optics was that of Nieto-Vesperinas *et al* [17] who used simulated annealing as an optimisation algorithm to improve image quality *after* the picture was taken in a telescope through the distorting atmosphere.

‡ In [4] and [7] the second feedback scheme is a little closer to the annealing process, but only changes that improved the image sharpness were kept; hence the crucial element of the possibility of choosing locally less favoured states that lead to globally preferable solutions is missing.

§ The search for ground states of glassy phases of spin models is another case where multiple minima abound. Even within a simulated annealing approach, the problem of determination of the ground state of a spin glass is a complex problem. Glassy phases are caused by the frustration of spins that are given ambiguous orientation instructions by their local environments. Local minima problems are expected to occur in SOS models when there are many surface steps, for example on sloped crystal surfaces, and in certain crystal growth problems. Here there are multiple choices for boundary location, which cause the systems to exhibit the large relaxation times characteristic of glassy systems. The relation between spin glasses and the model of (2) will be discussed in depth in a future paper.

beats of minimum length h_{max} and the $n\lambda/2\pi$ ambiguity is resolved. We verify that this is the real solution by using still another wavelength to measure the sharpness (3).

A second source of local minima in the telescope problem is related to the long-range nature of the interactions. Widely separated wavefront elements may be mutually phased, but interfere destructively with a second similar group. Such effects depend on Γ_{kl} which has a scale inversely proportional to the source size. it might seem that using a larger source would avoid this problem, but on balance the annealing together with a small source assures a long range quality of the phased mirror, and thus a minimum of the accumulated errors.

We have undertaken extensive simulations of different mirror arrays starting with a plane incident wavefront and random mirror heights. The mirror surfaces are kept parallel to some reference plane (i.e. they do not tilt). Full phasing was achieved on all samples studied, although the speed of convergence was reduced by such factors such as larger number of mirrors, wider band light, and more extended objects. We experimented with many different schedules for introduction of mirrors into the mosaic and for temperature variation. It was found that the optimal time for phasing was reached when a small seed group of mirrors was pointed towards the detector, while the rest pointed away. These were phased, and then additional mirrors were turned to the detector in small adjacent groups, and each group phased before another was turned†.

We began with a model of the Large Deployable Reflector [3], with its ninety hexagonal panels. Preliminary results for this model [18] were favourable, so the system was then enhanced to a larger model, of 16×16 mirrors in a square array. Approximately 6×10^5 mirror movements were required, on average, for the latter system to anneal to a perfect solution. This leads to timescales which are realistic (about ten minutes) for the correction of a large space telescope but not yet for atmospheric conditions.

We carried out tens of simulations and reached a phased solution in every case. Most of the variation between runs was due to different scheduling rather than to different initial conditions. Figure 1 shows results of a typical sequence of phasing, from a run where the temperature was reduced between iterations according to the schedule $T_{new}=0.995T_{old}$. A faster drop resulted in quenching into local minima, whereas a slower one resulted in too long a process. Each mirror was allowed 800 steps in the z direction, and the size of each step was 1/11th wavelength, with a total travel of some 73 wavelengths. Figure 1(a) corresponds to the initial state where the 4×4 central square is set to random heights and the outer ring is ignored. The graph shows a cross section through the speckle image associated with this configuration. After 1200 steps, the 4×4 array has annealed to a configuration containing several $n\lambda$ steps which nevertheless creates a diffraction limited image at λ_0 (figure 1(b)). After another 1000 steps, using the wavelength scheduling we achieve a configuration where the $n\lambda$ steps have disappeared; the diffraction limited image (b) is now obtained for any wavelength. Figure 1(c) corresponds to the stage where we have a central square of 6×6 mirrors in position and the next ring of 28 mirrors turned towards the detector

† In many of the simulations we used the more realistic choice of light of finite bandwidth, as opposed to an infinitely narrow band, the natural solution in the computer. This is achieved by adding a decay element which is closely related to a Lorentzian line shape to the cosine in (2):

$$\mathscr{H}=\sum_{k>l}|\Gamma_{kl}|^2\{1-\cos[2\pi((h_k-h_l)+\alpha\mathrm{i}|h_k-h_l|)/\lambda]\}$$

where $\alpha\approx0.0001$ determines the bandwidth.

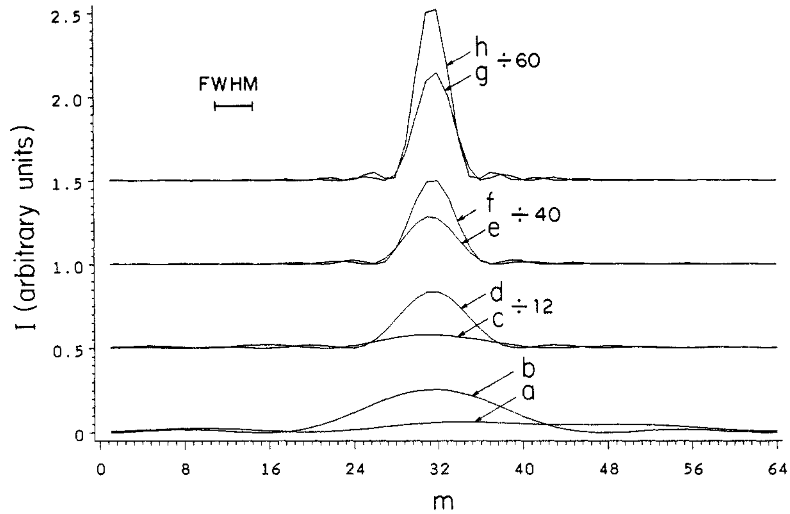

Figure 1. Intensity cross sections through the image at different stages in the adjustment of a 16×16 multi-mirror telescope. In (*a*) we illustrate the speckled image given by the initial central 4×4 configuration. In (*b*) we show the peak obtained after phasing this group. (*c*) and (*d*) as above for the 8×8 group, (*e*) and (*f*) for 12×12 and (*g*) and (*h*) for the full 16×16. The diffraction limited FWHM is shown by the bar. The baselines are shifted for clarity and the relative intensities of each pair of images are shown on the figure.

at random heights, and in the remaining graphs we see images corresponding to the phasings of the increasingly large central squares, each successful phasing being followed by the introduction of the next ring, until in figure 1(*h*) the entire telescope is phased. As the phased region has grown and the image improved, the sidelobes subside relative to the main peak. The central peak itself narrows as more and more panels are added to the calculation. Note that there are scale changes in the graphs, in order to bring out the detail in the earlier stages. Realisations of the mirror surface and three-dimensional images for this realisation are given in [19].

Our simulations clearly show that our algorithm can achieve optimal phasing within a time span that places realistic demands on both hardware and software. The method of Muller and Buffington [7], which corresponds to a rapid quench, obviously converges more quickly, but rarely reaches the optimal diffraction limited phasing solution. We plan to build a prototype model, which will be used to investigate the time gain achievable by using (3), and to explore scheduling questions further. It is intended that, in keeping with the analogy with the SOS system, such a prototype will also serve as an analogue computer to explore predictions for certain SOS models, and be used for the study of glassy phenomena. This approach also seems promising for correcting a microscope image when obtained through an inhomogeneous transparent sample.

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